Microwave Characterization of 2-D Random Materials: Numerical Simulations and Experiments

Thanh-Tuyen Nguyen and Geneviève Mazé-Merceur

***Abstract*—It is difficult to know what sample dimension allows us to characterize a heterogeneous material in the microwave range. To this end, in this paper, we emphasize the role of characteristic homogenization sizes for the determination of effective medium properties. We have performed a Monte Carlo (MC) numerical simulation of the reflection coefficient of a two-dimensional (2-D) plane composed of randomly distributed nonmagnetic conducting sticks. This method consists of determining the scattered field by integrating Maxwell's equations on a periodic realization of the plane, obtained by randomly dropping d sticks (per unit surface) on a square of side T periodically repeated in the plane. We show that the reflection coefficient strongly depends on the length introduced for the homogenization of this medium and tends to a steady value beyond a characteristic homogenization length T_c. We consider only the case where there is no current from one stick to another; in other words, there are only isolated sticks. After demonstrating the existence of a minimum homogenization size T_c, beyond which this sample can be considered as "homogenizable," we define an intrinsic parameter called the "square impedance." Measurements in the frequency range of 2–18 GHz have been carried out in free space in a broad-band lens focusing facility. Numerical results have been compared to the effective medium theory (EMT) results and validated by measurements.**

***Index Terms*— Broad-band lens focusing facility, conducting sticks, effective medium properties, heterogeneous material, measurement, microwave range, Monte Carlo simulation, random distribution, square impedance, 2-D screen.**

I. Introduction

THE design of composite materials with specific electromagnetic characteristics has led to the study of the properties of materials based on binary random mixtures.

The interest in two-dimensional (2-D) or quasi-2-D disordered materials appears in situations where additional physical constraints appear. For example, the situation requiring low-mass electromagnetic attenuators.

From the experimental point of view, the characterization of such a medium is of great importance in microwave applications. Most of the experimental methods are best suited for three-dimensional homogeneous isotropic materials. They provide accurate results for these materials in the microwave range. These methods are based on the measurement of reflection and transmission coefficients of a sample inserted in a transmission line [1] or free space [2]. Complex dielectric permittivity and complex magnetic permeability are then determined from these parameters.

The aim of this paper is to emphasize the role of characteristic sizes on the determination of the effective medium properties of 2-D random materials. In this paper, we present numerical, theoretical, and experimental results pertaining to the reflection coefficient of a 2-D plane of randomly distributed nonmagnetic conducting sticks. This medium is made of a screen containing metallic fibers and is illuminated by a plane wave. For this system, we expect a strong dependence of the scattering parameters versus the sample size. We will show that the reflection coefficient decreases with the sample size and tends to a steady value beyond a minimum homogenization size T_c. As a consequence, a sample of size $T > T_c$ can be considered as representative of the heterogeneous material.

At this stage, we need to define an intrinsic parameter of a 2-D material called square impedance. In fact, the material behavior cannot be described by the dielectric permittivity and material thickness.

Consequently, the main purpose of this paper is to demonstrate the existence of a characteristic homogenization size T_c, and to justify the use of the square impedance as the parameter intrinsic to the 2-D heterogeneous materials.

Here, we report on the numerical and theoretical results obtained by the STICK code and the effective medium theory (EMT). The numerical results have been given by Monte Carlo simulations (MCS's) of the electromagnetic response of stick systems with a fixed length T. Measurement results have been carried out in free space in a broad-band lens focusing facility. Numerical results have been compared to EMT and experiments in the frequency range of 2–18 GHz.

II. MCS of the Periodic T-Size Pattern Screen's Reflection Coefficient

We have written a code referred to as a "STICK code" in order to compute the electromagnetic response of a 2-D infinite plane composed of periodic T-size patterns of randomly distributed conducting sticks. This STICK code is coupled with an electromagnetic calculation [3]. It enables us: 1) to randomly generate a set of stick centers and their orientation angle at fixed density of sticks; 2) to mesh currents across fibers; 3) to determine the reflection and transmission coefficients for different polarizations after having taken into account

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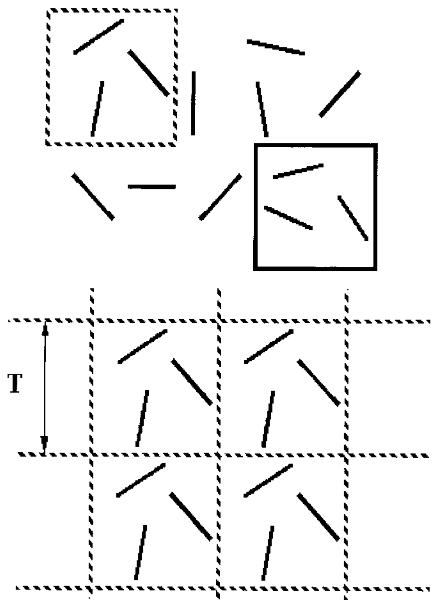

Fig. 1. Construction of the periodic pattern screen.

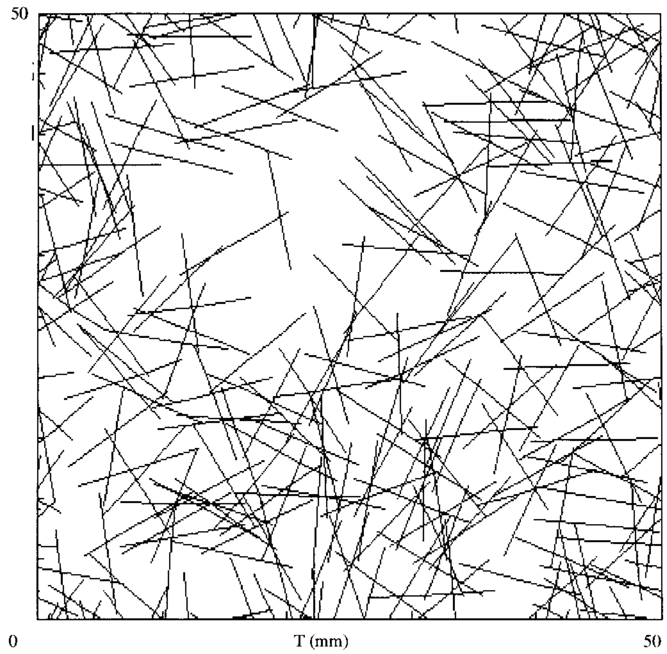

Fig. 2. Random distribution of sticks inside the pattern.

boundary nodes; and 4) to finally calculate the averaged reflection and transmission coefficients over the total number of MCS's.

Generally speaking, the "STICK code" corresponds to the MCS's of the periodic pattern screen's scattering parameters.

A. Random Generation of Sticks Within an Elementary Pattern

The conducting sticks are assumed to have the same length L, a conductivity s, and to be randomly distributed.

We consider only the case where there is no current from one stick to another. That is to say that there are only isolated conducting sticks.

The fundamental hypothesis is the following: the plane of sticks is approximated by a periodic network consisting of identical elementary patterns, as shown in Fig. 1.

Consider a sample which is a T-size square. d sticks (per surface unity) are randomly distributed in this square on sites, generated by the SP2 IBM computer (mainframe) [4]. An initial integer number called the seed is required for random generations. The next step is to attach a stick of length L to each site. We only retain sticks whose center position is inside the T-size square pattern.

In order to respect the boundary conditions, we take into account the sticks on the square sides. When a stick cuts any square side, the part of the stick located outside the square is added on the opposite parallel side, etc.

As an illustration, the sample used for the two-dimensional sticks system is shown in Fig. 2. The density is equal to $d = 0.1$ sticks/mm^2. The pattern size is equal to $T = 50$ mm. Sticks of equal length ($L = 10$ mm) are attached to the sites (stick centers) randomly distributed. For example, there are 250 sticks inside the square pattern, together with 75 periodic sticks, added according to boundary conditions of the structure.

The pattern is then reproduced in the two perpendicular directions of the plane, as shown in Fig. 1. The result is the generation of an infinite plane constituted by identical periodic patterns.

Once the computation of the sticks distribution is performed, we mesh the currents across fibers in accordance with an electromagnetic calculation, which is briefly presented below.

B. Calculation of the Screen's Electromagnetic Response

The electromagnetic calculation is used to compute the scattering of periodic structures composed of straight elements (perfectly conducting or not). The method used is the resolution of electric-field integral equations (EFIE). It is derived by enforcing the boundary condition that the tangential electric field on a conducting fiber surface is expressed as follows:

$$\vec{E}^t_{\text{inc}} + \vec{E}^t_{\text{scat}} = R\vec{J} \tag{1}$$

where $\vec{E}_{\text{inc}}$ denotes the incident electric field, $\vec{E}_{\text{scat}}$ the scattered field, $\vec{J}$ the current, R the resistance per unit length, and t the tangential component.

The scattered field in the presence of sticks can be expressed by the potentials as follows:

$$\vec{E}_{\text{scat}} = j\omega\vec{A} - \vec{\nabla}\Phi \tag{2}$$

with

$$\vec{A} = \mu \int_L G(\vec{r}, \vec{r}\,')\vec{J}(\vec{r}\,')\, d\vec{r}\,'$$

and

$$\Phi = \frac{1}{\varepsilon} \int_L G(\vec{r}, \vec{r}\,')\rho(\vec{r}\,')\, d\vec{r}\,'$$

where $\rho = -(1/j\omega)\,\vec{\nabla}\cdot\vec{J}$ denotes the charge density, $G(\vec{r}, \vec{r}\,')$ the periodic Green function of the system, $\vec{r}$ the observation point, and $\vec{r}\,'$ the source point.

The scattered fields are determined by Galerkin's method [5]. This method can be improved by choosing basis functions that more closely match the true current. The stick radius is considered as negligible compared to its length. Consequently, the stick current is axial. One set of basis functions which satisfy the conditions of continuity along the stick and zero-current at the stick ends is a series of triangular functions $\vec{f}_i$.

The application of Galerkin's method on (2) leads to

$$\langle \vec{f}_i, \vec{E}_{\text{inc}} \rangle = R\langle \vec{f}_i, \vec{J} \rangle - j\omega \langle \vec{f}_i, \vec{A} \rangle + \langle \vec{f}_i, \vec{\nabla}\Phi \rangle \qquad (3)$$

with

$$\langle \vec{f}_i, \vec{X} \rangle = \int_{\text{Segment}} \vec{f}_i \cdot \vec{X}\, dl$$

where a segment indicates integration over equal discretized stick segments. In this manner, we obtain the following set of moment-method equations to be solved numerically for the unknown currents C_n.

Once the coefficients C_n are known, the reflected and transmitted fields can be calculated according to Floquet's theorem [6]. This method allows us to determine the reflection and transmission coefficients for both copolarization and cross polarization.

C. Averaged Results

The reflection coefficient is averaged over the computation of a sufficient number of random configurations. Its variation as a function of the pattern size T will be studied at constant density and frequency.

III. Numerical Results and Comparison with the EMT

A. Numerical Results

Reflection and transmission coefficients are averaged over 50 Monte Carlo (MC) steps on an SP2 IBM computer. The stick length is 10 mm, its radius is 5 mm, and the external medium is assumed to be the vacuum. For $T = 50$ mm, the required central processing unit (CPU) calculation time for one disorder configuration is approximately 6 min and 4 h for respective stick densities of 0.01 and 0.1 sticks/mm^2.

The results computed at 5 GHz for the reflection and the transmission coefficients versus T, for densities equal to 0.01 and 0.1 sticks/mm^2, are shown in Fig. 3.

The averaged modulus of the reflection coefficient (in decibels) increases with the stick density and decreases with T up to a characteristic size T_c. It is worth noting that it tends to a steady value beyond a size T_c, which is approximately -31.77 and -10.87 dB for respective stick densities of 0.01 and 0.1 sticks/mm^2, as shown in Fig. 3. The homogenization length T_c depends on stick length, density, and frequency.

B. Comparison with the EMT

Applying the effective medium approach [7] to 2-D systems of randomly distributed sticks can lead to the predictions of the properties of these materials. This method consists of determining the expression for the scattering matrix at a given stick concentration. This method is limited to the quasi-static approximation. In other words, the stick size must remain low with respect to the wavelength of the incident field.

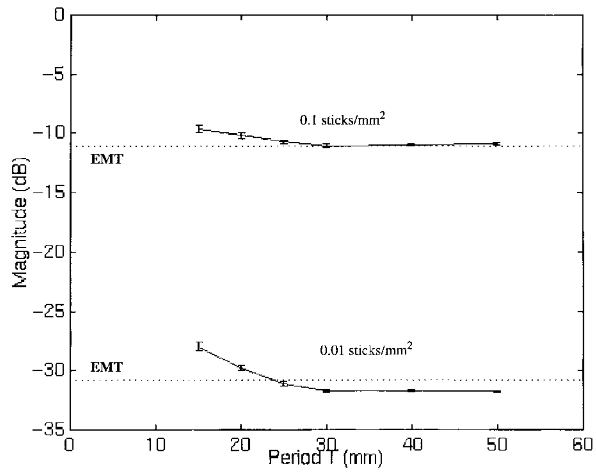

Fig. 3. Reflection coefficient versus T. Average of the modulus of the reflection coefficient computed over 50 MC simulations. The curves with error bars correspond to the values obtained with 0.1 sticks/mm^2 and with 0.01 sticks/mm^2 at 5 GHz, respectively. EMT values at 5 GHz are the two horizontal dotted lines. The stick length is 10 mm.

A conducting stick illuminated by an incident electromagnetic wave behaves as an oscillating dipole (at the same frequency as the incoming wave), which is considered "point-like." The geometry of the fiber will be taken into account by use of the depolarization factor. The plane of the sticks can be considered as a homogeneous current sheet with an applied incident field. This uniform current is directed along the incident field polarization and is proportional to it. A fiber is isolated and placed at the center of a cavity of radius r_0, which is defined by $d\pi r_0^2 = 1$ (d is the stick density). It means that the cavity contains approximately one dipole. The source at the origin is obtained in the quasi-static approximation, which is to say that the Foucault currents are neglected. Finally, the stick current is calculated by self-consistency equations. Results from this theory corresponding to $L = 10$ mm and $\varnothing = 10$ μm at 5 GHz are shown in Fig. 3. They are in good agreement with the results obtained by our computation [8].

IV. Square Impedance

In paragraph III, we have seen evidence of the existence of T_c beyond which a heterogeneous material sample can be considered as "homogenizable." We next investigate the validity of a parameter intrinsic to the 2-D materials, which we will call the "square impedance." It is derived from the field equations through the material interface. We present two solutions to determine it.

The first consists of studying the whole system (stick material + spacer of thickness h + short circuit) which is a generalized Salisbury screen (see Fig. 4). The normalized square impedance can be derived from the reflexion coefficient S_{11cc} of the following system:

$$z_{\text{square}} \equiv \frac{Z_{\text{square}}}{Z_0} = -\frac{1}{2}\,\frac{(1-e^{2jkh})(1+S_{11cc})}{S_{11cc}+e^{2jkh}} \qquad (4)$$

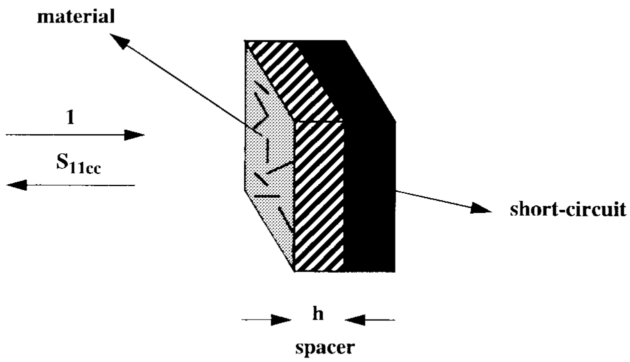

Fig. 4. Generalized Salisbury screen.

where k is the wavenumber in the spacer material and $Z_0 = 120\pi\ \Omega$.

The second method enables us to calculate the normalized square impedance from the reflection coefficient of the stick plane S_{11} (derived from the scattering matrix) by

$$z_{\text{square}} = -\frac{1}{2}\left(1 + \frac{1}{S_{11}}\right). \quad (5)$$

Consequently, the square impedance can be determined in the following ways:

1) from the reflection coefficient computed by the "STICK code";
2) from the calculation based on the EMT;
3) from the measurements of the reflection coefficient in the transmission/reflection configuration;
4) from the measurement of the reflection coefficient with a spacer and short circuit.

V. Experimental Validations

A. Experimental Setup [9]

Measurements are performed in free space in a lens focusing facility, as shown in Fig. 5.

B. Materials Studied

The materials studied are as follows.

The first one corresponds to paper $A1$, containing a host material of polyethylene pulp ($\varepsilon_r = 1.5 - j0$), and conducting sticks. The thickness of these materials is approximately 1 mm.

The remaining test materials consist of silica tissues (tissues $B1$ and $B2$), containing conducting sticks deposited on a support Depron material ($\varepsilon_r = 1.3 - j0$).

C. Validations

The measurements of the reflection coefficient of paper $A1$ have been carried out in the broad-band lens focusing facility and are compared with calculations by the STICK code and EMT (see Fig. 6). In Fig. 7, we observe a good agreement for the square impedance derived from the computed and measured reflection coefficient, and from the EMT, called z_{square}(STICK code), $z_{\text{square}}(S_{11\text{measured}})$, and z_{square}(EMT), respectively. The difference between the experimental and theoretical curves is likely due to: 1) the host material thickness,

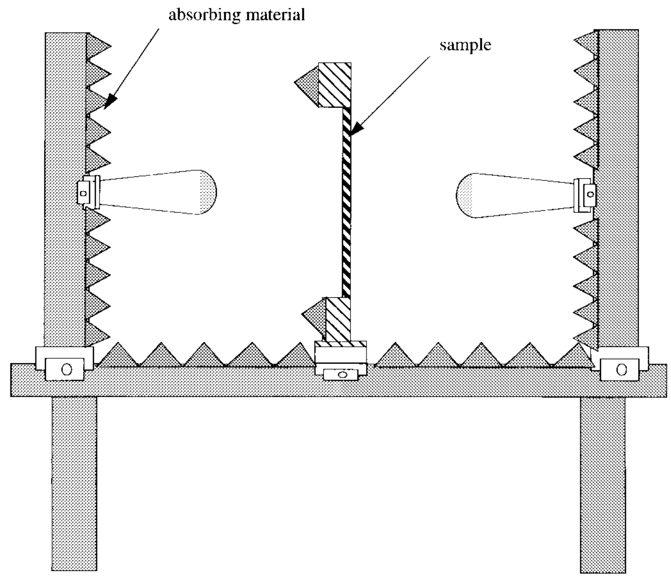

Fig. 5. Lens focusing facility in the transmission/reflection configuration with broad-band horn antenna (2–18 GHz).

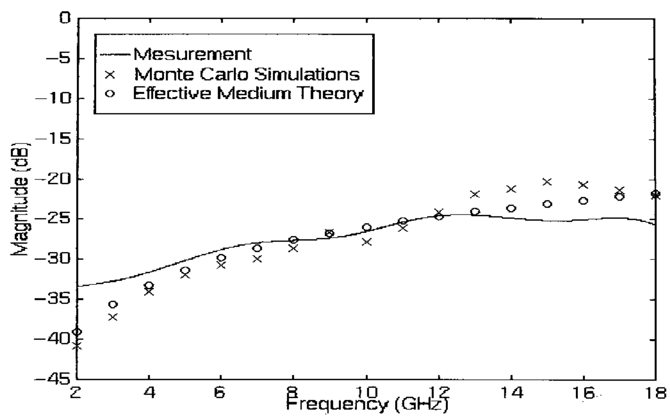

Fig. 6. Reflection coefficient. Numerical (STICK code), theoretical (EMT), and experimental reflection coefficients of paper $A1$.

which is obviously not zero, but is approximately 1 mm; 2) the measurement limitations, such as the low noise-to-signal ratio, edge scattering and difficulty in positioning the sample (low thickness of samples); and 3) the fact that the hypotheses used for the STICK code are not quite the same as the measurement data. For example, the external medium is assumed to be the vacuum, whereas the permittivity of the host material is more than $1.0 - j0$ and the measured density is not very accurate. The difference may also come from the hypothesis that no current passes from one stick to another. This point is discussed in the following paragraphs.

As far as the tissues $B1$ and $B2$ are concerned, the fiber resonance phenomenon is illustrated for $L = 30$ mm in Fig. 8, respectively, by plotting the real and imaginary parts of the reflection coefficient versus frequency; the former presents a maximum and the latter vanishes at the resonant frequency. Note that when the stick diameter is much less than its length, the resonant frequency is defined so that the half-wavelength $\lambda/2$ is equal to the stick length. We observe in Fig. 7 that the experimental resonant frequency is less than the theoretical

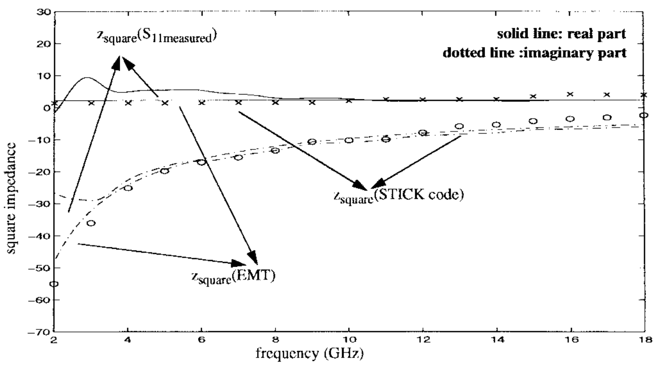

Fig. 7. Square impedance derived from the numerical (STICK code), theoretical (EMT), and experimental results of the reflection coefficient of paper $A1$. The dotted lines correspond to imaginary parts and the solid lines to real parts of the square impedance. The x-mark represent its real part and the circles its imaginary part obtained by the STICK code.

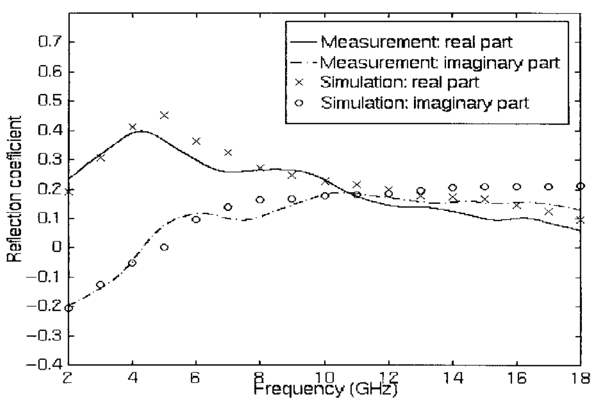

Fig. 8. Resonance behavior of fibers of tissue $B1$.

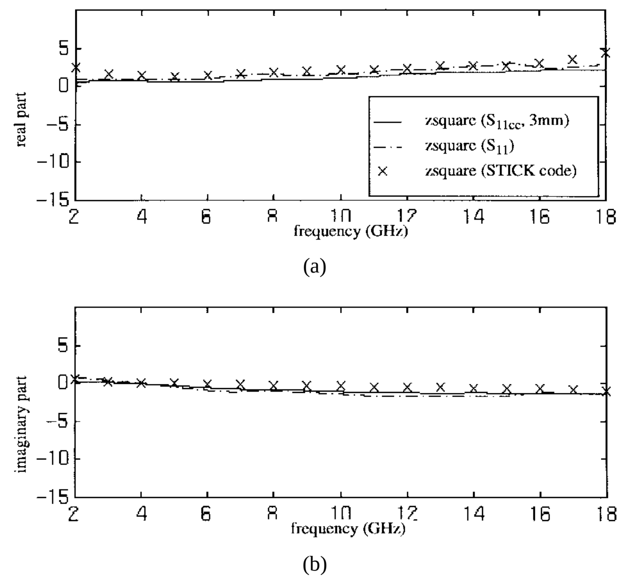

Fig. 9. Square impedance. Experimental validation of $z_{\rm square}$ of tissue $B1$.

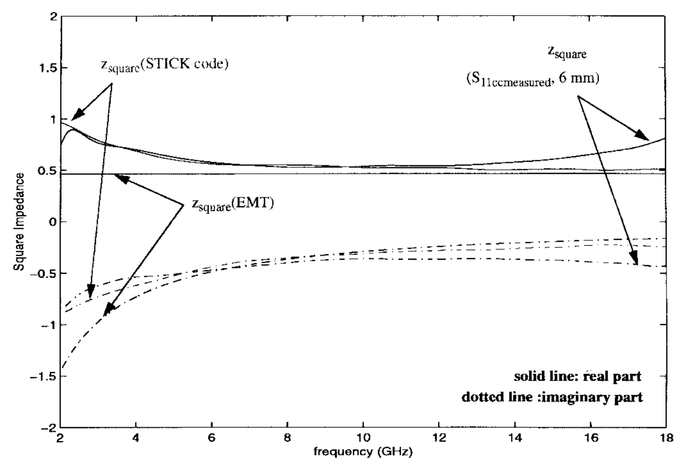

Fig. 10. Square impedance of tissue $B2$. The dotted lines correspond to imaginary parts and the solid lines to real parts of the square impedance.

one corresponding to the isolated fibers of the same length. Measurements and numerical results are mentioned in Fig. 7; the experimental resonance is at 4.2 GHz and the theoretical one at 5 GHz (tissue B2). This displacement of the resonance toward low frequencies is due to the host material (whose permittivity is greater than $1.0 - j0$, permittivity assumed in the calculations), to the air gaps between tissue and support material, and to the fact that fibers may overlap and form clusters whose "equivalent" length is certainly higher than that of the isolated fibers. Results show a good agreement between numerical and experimental values.

We represent the intrinsic parameter of these tissues in Figs. 9 and 10, respectively, corresponding to tissue $B1$ and the second one to tissue $B2$. Note that $z_{\rm square}$ [$S_{11cc{\rm measured}}$, 3 mm (or 6 mm)] represents the square impedance calculated from the measurement of the reflection coefficient of the system (tissue + 3 mm (or 6 mm)-thickness spacer + short circuit). We observe that the imaginary part of the square impedance calculated from the EMT stands aside from the measurements for the tissue $B2$ ($L = 5$ mm). We recall that in the EMT, fibers are assumed not to overlap. Since the hypothesis which states that there is no interaction between fibers is not quite true for strong densities, fibers may form clusters whose "equivalent" length is greater than the isolated fiber length. In consequence, if the stick length varies from 5 to 10 mm, we find good agreement between the EMT and measurements for an "equivalent" length of $L = 7$ mm, whose value is taken for the curve $z_{\rm square}$(EMT) shown in Fig. 10. However, the real part of the square impedance remains relatively insensitive to the variation of the stick length.

VI. Conclusion

We have developed a numerical simulation in the microwave range of the electromagnetic behavior of a 2-D random-sticks material consisting of identical periodic T-size patterns, and have shown that the reflection coefficient depends strongly on the sample size up to T_c. Beyond T_c, a sample of the heterogeneous material can be considered as "homogenizable" for fixed density, frequency, and stick length, and characterized by its square impedance. The square impedance can be determined by the measurement of the reflection coefficient of the plane of fibers. Alternatively, it can be derived from the measurement of the reflection coefficient of the fiber plane positioned in front of a spacer and short circuit. The resonance behavior of the conducting fibers illuminated by an incident electromagnetic wave is exhibited by the numerical and experimental results in the range of 2–18 GHz. Moreover, numerical results agree well with the EMT and measurements.

Future work should be carried out on the study of T_c versus frequency and density. The characteristic sample size in the static limit corresponds to the correlation length of the clusters in the system. Work is currently in progress to determine the dependence of the size T_c on the stick concentration d. An important limitation to this work is the time needed for computation at higher concentrations.

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REFERENCES

[1] J. Baker-Jarvis, E. J. Vanzura and W. A. Kissick, "Improved technique for determining complex permittivity with the transmission/reflection method," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1096–1103, Aug. 1990.
[2] A. Priou, "Dielectric properties of heterogeneous materials," *Piers 6*. Amsterdam, The Netherlands: Elsevier, 1992.
[3] J. M. Sajer, "Code FSS3D," CEA-DAM, Le Barp, France, Internal Rep., 1994.
[4] I. Balberg, "Computer study of the percolation threshold in a two-dimensional anisotropic system of conducting sticks," *Phys. Rev. B, Condens. Matter*, vol. 28, no. 7, pp. 3799–3812, 1983.
[5] M. Sadifu, *Numerical Techniques in Electromagnetics*. Boca Raton, FL: CRC Press, 1992.
[6] J. J. H. Wang, *Generalized Moment Methods in Electromagnetics. Formulation and computer Solution of Integral Equations.* New York: Wiley, 1991.
[7] M. Barthélémy, T. T. Nguyen, and P. G. Zérah, "Effective medium theory and Monte Carlo simulations for the reflection coefficient of planar fiber composites," *Phys. Rev. Lett.*, submitted for publication.
[8] T. T. Nguyen, J. M. Sajer, and J. Pommier, "Electromagnetic behavior of two-dimensional systems of randomly distributed conducting sticks," in *IEEE ICSD'95 Congr.*, Leicester, U.K., July 1995.
[9] D. K. Ghodgaondar, V. V. Varadan, and V. K. Varadan, "Free-space measurement of complex permittivity and complex permeability of magnetic materials at microwave frequencies," *IEEE Trans Instrum. Meas.*, vol 39, pp. 387–394, Apr. 1990.

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